Computational Physics – Wiederholungs-Klausur

31.03.2016

Instructions: Please answer all 6 questions. You can answer in one of the following languages: German, English, Italian, Spanish, French, or Portuguese.

Question 1: Generation of Random Numbers

(a) Explain how pseudo random numbers can be generated on a computer using the linear congruent method. What are the properties of the thus generated random numbers?

(b) Random number generators as implemented in most libraries typically generate (pseudo-) random numbers x following a uniform probability distribution $u(x) = 1$ in the interval [0,1]. However, often one needs random numbers that follow some non-uniform probability distribution $f(x)$. Which two methods do you know for generating random numbers x following an *arbitrary* probability distribution $f(x)$? Explain both methods and sketch the algorithms.

(c) Which of the two methods would be best suited for generating random numbers x following a linear distribution $f(x) = 2x$ for $x \in [0, 1]$ and why?

Question 2: Interpolation and Differentiation

(a) Explain the method of interpolation by (cubic) splines. What is the basic idea? What are the properties of the interpolating spline functions (for example with regard to continuity (Stetigkeit) of the function and its derivatives)? What is the advantage over Lagrange interpolation?

(b) Numerical Differentiation: Explain the Richardson extrapolation scheme. What is the advantage over simpler numerical differentiation schemes?

Question 3: Monte-Carlo Integration

(a) How is the integral of some function $f(x)$in the interval [a,b] approximated by Monte-Carlo (MC) importance sampling? Sketch the algorithm for evaluating $\int_a^b f(x)dx$ by MC importance sampling.

(b) What is the advantage of importance sampling over simple Monte-Carlo integration?

(c) For a d-dimensional integral $\int_{a_1}^{b_1} \dots \int_{a_d}^{b_d} f(x_1, \dots x_d)dx_1 \dots dx_d$, how does the error of MC integration scale with the number of evaluation points? From which dimension d on, MC integration becomes more efficient than other numerical integration methods?

Question 4: Ordinary Differential Equations

(a) Explain the Euler and the Runge-Kutta methods to solve ordinary differential equations, and discuss their advantages and disadvantages.

(b) Consider the initial value problem $\frac{t}{2y'+2} = 3y$, with $y(0)=1$. Estimate the value of $y(0.02)$ using the Euler method and a step size of $h=0.01$.

(c) Sketch a program to obtain the value $y(1)$ using the 4th order Runge-Kutta method.

Question 5: Integration

(a) **Derive the trapezoidal rule** for integrating a function in the interval $[x, x+\Delta x]$ for a small value of Δx, and show how to apply this rule to integrate a function in the (large) interval $[a,b]$.

(b) Sketch a program that integrates the function $1/\sqrt{x}$ in the interval $[-1,1]$ using the trapezoidal method. What could you do to decrease the error in the evaluation of the integral.

Question 6: Root finding

(a) Using the Newton-Raphson method, find analytically an approximation to the value of $\sqrt{2}$ starting with $x_0=2$. Show that this value, after 2 iterations is given by $17/12 \approx 1.4167$

(b) Repeat the previous question with the bisection and secant methods. Use as starting interval $[1,2]$. Comment on the results and the errors obtained.